LOGARITMOS

Problema 120:

Calcular, sin empleo de tablas, el logaritmo de la siguiente expresión:

$$x = \frac{3{,}2^3 \cdot 0{,}64^5}{0{,}0125 \cdot \sqrt[4]{80^3}}$$

$$\log 2 = 0{,}301030$$

Solución Problema 120:

$$x = \frac{3{,}2^3 \cdot 0{,}64^5}{0{,}0125 \cdot \sqrt[4]{80^3}}$$

Tomamos logaritmos:

$$logx = log\frac{3{,}2^3 \cdot 0{,}64^5}{0{,}0125 \cdot \sqrt[4]{80^3}} = log\frac{(\frac{32}{10})^3 \cdot (\frac{64}{100})^5}{\frac{125}{10000} \cdot 80^{\frac{3}{4}}} = \log[(\frac{2^4 \cdot \cancel{2}}{5 \cdot \cancel{2}})^3 \cdot (\frac{2^4 \cdot \cancel{2^2}}{\cancel{2^2} \cdot 5^2})^5] - \log\left[\frac{\cancel{5^3}}{2^4 \cdot 5 \cdot \cancel{5^3}} \cdot (2^4 \cdot 5)^{\frac{3}{4}}\right] =$$

$$= \log\left[\left(\frac{2^{12}}{5^3}\right) \cdot \left(\frac{2^{20}}{5^{10}}\right)\right] - \log\left[\left(\frac{1}{2^4 \cdot 5}\right) \cdot (2^3 \cdot 5^{\frac{3}{4}})\right] = \log\left(\frac{2^{12}}{5^3}\right) + log(\frac{2^{20}}{5^{10}}) - [\log\left(\frac{1}{2^4 \cdot 5}\right) + log(2^3 \cdot 5^{\frac{3}{4}})] =$$

$$= log2^{12} - log5^3 + log2^{20} - log5^{10} - [\log 1 - \log(2^4 \cdot 5) + log2^3 + log\ 5^{\frac{3}{4}}] =$$

$$= \log 2^{12} - log5^3 + \log 2^{20} - log5^{10} - \{\log 1 - [\log 2^4 + log5] + log2^3 + log\, 5^{\frac{3}{4}}\} =$$

$$= \log 2^{12} - log5^3 + \log 2^{20} - log5^{10} - \log 1 + [\log 2^4 + log5] - log2^3 - log\, 5^{\frac{3}{4}} =$$

$$= \log 2^{12} - log5^3 + \log 2^{20} - log5^{10} - \log 1 + \log 2^4 + log5 - log2^3 - log\, 5^{\frac{3}{4}} =$$

$$= \log 2^{12} + \log 2^{20} + \log 2^4 - log2^3 - log5^3 - log5^{10} - log\, 5^{\frac{3}{4}} + log5 - \log 1 =$$

$$= 12\log 2 + 20log2 + 4\log 2 - 3log2 - 3log5 - 10log5 - \frac{3}{4}log\, 5 + log5 - \log 1 =$$

$$= 12\log 2 + 20log2 + 4\log 2 - 3log2 - 3log5 - 10log5 - \frac{3}{4}log\, 5 + log5 - \log 1 = 33log2 - \frac{51}{4}log5 - 0 =$$

$$= 33log2 - \frac{51}{4}log\frac{10}{2} = 33\log 2 - \frac{51}{4}(\log 10 - log2) = 33\log 2 - \frac{51}{4}\log 10 + \frac{51}{4}log2 = \frac{183}{4}log2 - \frac{51}{4}log10 =$$

$$\frac{183}{4}\cdot 0{,}301030 - \frac{51}{4}\cdot 1 = \frac{55{,}08849}{4} - \frac{51}{4} = \frac{55{,}08849 - 51}{4} = \frac{4{,}08849}{4} = \mathbf{1{,}0221225}$$