PROGRESIONES GEOMÉTRICAS

Problema 66:

Formar una progresión geométrica de cuatro términos si el segundo es 20 y la suma de los cuatro términos es 425.

Solución problema 66:

Sabemos que:

$a_1 + a_2 + a_3 + a_4 = 425$

$a_2 = 20$

Ponemos todos los términos en función de a_2:

$\frac{a_2}{r} + a_2 + a_2 \cdot r + a_2 \cdot r^2 = 425$

$a_2 + a_2 \cdot r + a_2 \cdot r^2 + a_2 \cdot r^3 = 425 \cdot r$

Sabemos que:

$a_2 = 20$

Luego,

$20 + 20r + 20r^2 + 20r^3 = 425r$

$20r^3 + 20r^2 + 20r - 425r + 20 = 0$

$20r^3 + 20r^2 - 405r + 20 = 0$

Aplicando Ruffini:

	20	20	-405	20
_4		80	400	-20
	20	100	-5	0

Queda la ecuación de 2° grado:

$20r^2 + 100r - 5 = 0$

Simplificando por 5:

$4r^2 + 20r - 1 = 0$

$$r = \frac{-20 \pm \sqrt{400 + 16}}{8} = \frac{-20 \pm \sqrt{416}}{8} = \frac{-20 \pm 4\sqrt{26}}{8} = \frac{-5 \pm \sqrt{26}}{2}$$

$$r_2 = \frac{-5 + \sqrt{26}}{2}$$

$$r_3 = \frac{-5 - \sqrt{26}}{2}$$

Para r= 4:

$$a_1 = \frac{a_2}{r} = \frac{20}{4} = 5$$

$a_2 = 20$

$a_3 = a_2 \cdot r = 20 \cdot 4 = 80$

$a_4 = a_2 \cdot r^2 = 20 \cdot 16 = 320$

Para r_2:

$$r_2 = \frac{-5 + \sqrt{26}}{2}$$

$$a_1 = \frac{a_2}{r} = \frac{20}{\frac{\sqrt{26} - 5}{2}} = \frac{40}{\sqrt{26} - 5} = \frac{40}{\sqrt{26} - 5} \cdot \frac{\sqrt{26} + 5}{\sqrt{26} + 5} = 40(\sqrt{26} + 5)$$

$a_2 = 20$

$$a_3 = a_2 \cdot r = 20 \cdot \frac{\sqrt{26} - 5}{2} = 10(\sqrt{26} - 5)$$

$$a_4 = a_3 \cdot r = 10(\sqrt{26} - 5) \cdot \frac{\sqrt{26} - 5}{2} = 5(51 - 10\sqrt{26})$$

Para r_3:

$$r_3 = \frac{-5-\sqrt{26}}{2}$$

$$a_1 = \frac{a_2}{r} = \frac{20}{\frac{-\sqrt{26}-5}{2}} = \frac{40}{-\sqrt{26}-5} = \frac{40}{-\sqrt{26}-5} \cdot \frac{\sqrt{26}-5}{\sqrt{26}-5}$$

$$a_1 = -40(\sqrt{26}-5)$$

$$a_2 = 20$$

$$a_3 = a_2 \cdot r = 20 \cdot \frac{-\sqrt{26}-5}{2} = -10(\sqrt{26}+5)$$

$$a_4 = a_3 \cdot r = -10\left(\sqrt{26}-5\right) \cdot \frac{\sqrt{26}-5}{2} = -5(51-10\sqrt{26})$$