COMBINATORIA

<u>Problema 86</u>:

Resolver la siguiente ecuación:

$$\binom{x}{0} + \binom{x}{1} + \binom{x}{2} + \binom{x}{3} = \frac{x(x^2 + 6)}{6}$$

Solución Problema 86:

Sabemos que

$$\binom{x}{0} = C_{x,0} = \frac{x!}{0! \, (x! - 0!)} = \frac{x!}{1 \cdot (x - 0)!} = \frac{x!}{x!} = 1$$

Luego:

$$1 + x + \frac{x(x - 1)}{2 \cdot 1} + \frac{x \cdot (x - 1) \cdot (x - 2)}{3 \cdot 2 \cdot 1} = \frac{x(x^2 + 6)}{6}$$

$$1 + x + \frac{x(x - 1)}{2} + \frac{x \cdot (x - 1) \cdot (x - 2)}{6} = \frac{x(x^2 + 6)}{6}$$

$$6 + 6x + 3x(x - 1) + x \cdot (x - 1) \cdot (x - 2) = x(x^2 + 6)$$

$$6 + 6x + 3x^2 - 3x + x \cdot (x^2 - x - 2x + 2) = x(x^2 + 6)$$

$$6 + 6x + 3x^2 - 3x + x \cdot (x^2 - 3x + 2) = x(x^2 + 6)$$

$$6 + 6x + 3x^2 - 3x + x^3 - 3x^2 + 2x = x^3 + 6x$$

$$6 + 6x + 3x^2 - 3x + x^3 - 3x^2 + 2x - x^3 - 6x = 0$$

$$6 - x = 0$$

$$-x = -6$$

$$x = 6$$