PROBLEMAS DE TRIGONOMETRÍA

Problema 188:

Demostrar que se verifica la siguiente igualdad:

$$\frac{(1+tg\ a)^2}{1+\ tg^2a}-\frac{\dfrac{tg\ a}{1-tg\ a}+\dfrac{tg\ a}{1+tg\ a}}{\dfrac{sec^2a\cdot\ cosec^2a}{cosec^2a-\ sec^2a}}=1$$

Solución Problema 188:

$$\frac{(1+tg\ a)^2}{1+\ tg^2a}-\frac{\dfrac{tg\ a}{1-tg\ a}+\dfrac{tg\ a}{1+tg\ a}}{\dfrac{sec^2a\cdot\ cosec^2a}{cosec^2a-\ sec^2a}}=\frac{(1+tg\ a)^2}{1+\ tg^2a}-\frac{\dfrac{tg\ a(1+tg\ a)+tg\ a(1-tg\ a)}{1-\ tg^2a}}{\dfrac{sec^2a\cdot\ cosec^2a}{cosec^2a-\ sec^2a}}=$$

$$=\frac{(1+tg\ a)^2}{1+\ tg^2a}-\frac{\dfrac{tg\ a+tg^2a+tg\ a-tg^2a}{1-\ tg^2a}}{\dfrac{sec^2a\cdot\ cosec^2a}{cosec^2a-\ sec^2a}}=\frac{(1+tg\ a)^2}{1+\ tg^2a}-\frac{\dfrac{2tg\ a}{1-\ tg^2a}}{\dfrac{sec^2a\cdot\ cosec^2a}{cosec^2a-\ sec^2a}}=\frac{(1+tg\ a)^2}{1+\ tg^2a}-\frac{tg\ 2a}{\dfrac{sec^2a\cdot\ cosec^2a}{cosec^2a-\ sec^2a}}$$

Para mayor claridad haremos aparte el denominador:

$$\frac{sec^2a\cdot\ cosec^2a}{cosec^2a-\ sec^2a}=\frac{\dfrac{1}{cos^2a}\cdot\dfrac{1}{sen^2a}}{\dfrac{1}{sen^2a}-\dfrac{1}{cos^2a}}=\frac{\dfrac{1}{cos^2a\cdot\ sen^2a}}{\dfrac{cos^2a-\ sen^2a}{sen^2a\cdot\ cos^2a}}=\frac{1}{cos^2a-\ sen^2a}=\frac{1}{\cos 2a}$$

Luego, sustituyendo los cálculos hechos tenemos que:

$$\frac{(1+tg\ a)^2}{1+\ tg^2a}-\frac{tg\ 2a}{\frac{sec^2a\cdot\ cosec^2a}{cosec^2a-\ sec^2a}}=\frac{(1+tg\ a)^2}{1+\ tg^2a}-\frac{tg\ 2a}{\frac{1}{\cos 2a}}=\frac{(1+tg\ a)^2}{1+\ tg^2a}-\frac{\frac{sen\ 2a}{\cos 2a}}{\frac{1}{\cos 2a}}=\frac{(1+tg\ a)^2}{1+\ tg^2a}-sen\ 2a=$$

$$=\frac{1+\ tg^2a+2tg\ a}{1+\ tg^2a}-sen\ 2a=\frac{1+\ tg^2a}{1+\ tg^2a}+\frac{2tg\ a}{1+\ tg^2a}-sen\ 2a=1+\frac{2tg\ a}{sec^2a}-sen\ 2a=1+\frac{\frac{2sen\ a}{\cos a}}{\frac{1}{cos^2a}}-sen\ 2a=$$

$$1+\frac{2sen\ a\cdot cos^2a}{\cos a}-sen\ 2a=1+2sen\ a\cdot\cos a-sen\ 2a=1+sen\ 2a-sen\ 2a=1$$