

PROBLEMAS DE TRIGONOMETRÍA

Problema 167:

$$\operatorname{sen} a = \frac{1}{2}\sqrt{2 - \sqrt{2}}$$

Calcula el valor de $\operatorname{tg} 2a$

Solución Problema 167:

Para ello,

$$\operatorname{tg} 2a = \frac{\operatorname{sen} 2a}{\cos 2a}$$

Hallamos $\operatorname{sen}^2 a$

$$\operatorname{sen} a = \frac{1}{2}\sqrt{2 - \sqrt{2}}$$

$$\operatorname{sen}^2 a = \left(\frac{1}{2}\sqrt{2 - \sqrt{2}}\right)^2 = \frac{1}{4} \cdot (2 - \sqrt{2}) = \frac{2 - \sqrt{2}}{4}$$

$$\operatorname{sen}^2 a = \frac{2 - \sqrt{2}}{4}$$

Hallamos $\cos^2 a$:

$$\cos^2 a = 1 - \sin^2 a = 1 - \frac{2 - \sqrt{2}}{4} = \frac{4 - 2 + \sqrt{2}}{4} = \frac{2 + \sqrt{2}}{4}$$

$$\cos^2 a = \frac{2 + \sqrt{2}}{4}$$

Hallamos $\cos a$:

$$\cos a = \sqrt{\frac{2 + \sqrt{2}}{4}} = \frac{1}{2} \sqrt{2 + \sqrt{2}}$$

Sabemos que:

$$\sin 2a = 2 \sin a \cdot \cos a$$

Y

$$\cos 2a = \cos^2 a - \sin^2 a$$

Luego:

$$\operatorname{tg} 2a = \frac{\sin 2a}{\cos 2a} = \frac{2 \sin a \cdot \cos a}{\cos^2 a - \sin^2 a} = \frac{2 \cdot \left[\frac{1}{2} \sqrt{2 - \sqrt{2}} \right] \cdot \left[\frac{1}{2} \sqrt{2 + \sqrt{2}} \right]}{\frac{2 + \sqrt{2}}{4} - \frac{2 - \sqrt{2}}{4}} = \frac{2 \cdot \frac{1}{4} \left[\sqrt{(2 - \sqrt{2}) \cdot (2 + \sqrt{2})} \right]}{\frac{2 + \sqrt{2}}{4} - \frac{2 - \sqrt{2}}{4}} =$$

$$= \frac{\frac{1}{2}[\sqrt{4-2}]}{\frac{2+\sqrt{2}-2+\sqrt{2}}{4}} = \frac{\frac{1}{2}\sqrt{2}}{\frac{2\sqrt{2}}{4}} = \frac{\frac{1}{2}\sqrt{2}}{\frac{1\sqrt{2}}{2}} = 1$$