

## PROBLEMAS DE TRIGONOMETRÍA

Problema 151:

Escribir la siguiente expresión en función de “cos a” solamente:

$$\left( \frac{\operatorname{tg} \frac{a}{2}}{\operatorname{sen} a} - 2\operatorname{sen}^2 \frac{a}{2} \right) \cdot \cos^2 \frac{a}{2}$$

Solución Problema 151:

$$\left( \frac{\operatorname{tg} \frac{a}{2}}{\operatorname{sen} a} - 2\operatorname{sen}^2 \frac{a}{2} \right) \cdot \cos^2 \frac{a}{2} = \left( \frac{\frac{\operatorname{sen} \frac{a}{2}}{\cos \frac{a}{2}}}{\operatorname{sen} a} - 2\operatorname{sen}^2 \frac{a}{2} \right) \cdot \cos^2 \frac{a}{2} =$$

$$\left( \frac{\operatorname{sen} \frac{a}{2}}{\operatorname{sen} a \cdot \cos \frac{a}{2}} - 2\operatorname{sen}^2 \frac{a}{2} \right) \cdot \cos^2 \frac{a}{2} = \frac{\operatorname{sen} \frac{a}{2} \cdot \cos^2 \frac{a}{2}}{\operatorname{sen} a \cdot \cos \frac{a}{2}} - 2\operatorname{sen}^2 \frac{a}{2} \cdot \cos^2 \frac{a}{2} =$$

$$= \frac{\operatorname{sen} \frac{a}{2} \cdot \cos \frac{a}{2}}{\operatorname{sen} a} - 2\operatorname{sen}^2 \frac{a}{2} \cdot \cos^2 \frac{a}{2} =$$

$$= \frac{\operatorname{sen} \frac{a}{2} \cdot \cos \frac{a}{2}}{\operatorname{sen} a} - 2\operatorname{sen} \frac{a}{2} \cdot \operatorname{sen} \frac{a}{2} \cdot \cos \frac{a}{2} \cdot \cos \frac{a}{2} =$$

$$= \frac{\operatorname{sen} \frac{a}{2} \cdot \operatorname{cos} \frac{a}{2}}{\operatorname{sen} a} - 2 \operatorname{sen} \frac{a}{2} \cdot \operatorname{cos} \frac{a}{2} \cdot \operatorname{sen} \frac{a}{2} \cdot \operatorname{cos} \frac{a}{2} =$$

Sabemos que:

$$2 \operatorname{sen} x \cdot \operatorname{cos} x = \operatorname{sen} 2x$$

Luego:

$$\begin{aligned} &= \frac{\operatorname{sen} \frac{a}{2} \cdot \operatorname{cos} \frac{a}{2}}{\operatorname{sen} a} - \operatorname{sen} a \cdot \operatorname{sen} \frac{a}{2} \cdot \operatorname{cos} \frac{a}{2} = \\ &= \frac{\operatorname{sen} \frac{a}{2} \cdot \operatorname{cos} \frac{a}{2} - \operatorname{sen}^2 a \cdot \operatorname{sen} \frac{a}{2} \cdot \operatorname{cos} \frac{a}{2}}{\operatorname{sen} a} = \frac{\operatorname{sen} \frac{a}{2} \cdot \operatorname{cos} \frac{a}{2} (1 - \operatorname{sen}^2 a)}{\operatorname{sen} a} = \\ &= \frac{\operatorname{sen} \frac{a}{2} \cdot \operatorname{cos} \frac{a}{2} \cdot \operatorname{cos}^2 a}{\operatorname{sen} a} = \end{aligned}$$

Multiplicando numerador y denominador por 2:

$$= \frac{2 \operatorname{sen} \frac{a}{2} \cdot \operatorname{cos} \frac{a}{2} \cdot \operatorname{cos}^2 a}{2 \operatorname{sen} a} =$$

Sabemos que:

$$2 \operatorname{sen} x \cdot \operatorname{cos} x = \operatorname{sen} 2x$$

Luego:

$$= \frac{\text{sen } a \cdot \cos^2 a}{2 \text{sen } a} = \frac{\cos^2 a}{2}$$