PROBLEMAS DE TRIGONOMETRÍA

Problema 6:

Sabiendo que

$$sena = \frac{1}{4}(\sqrt{6} - \sqrt{2})$$

Calcular el valor de cotg 2a

Solución Problema 6:

Previamente calculamos el valor de $\mathrm{sen}^{\mathbf{2}}\mathrm{a}$

$$sena = \frac{1}{4}(\sqrt{6} - \sqrt{2})$$

$$\mathrm{sen}^2\mathrm{a} = [\frac{1}{4}(\sqrt{6} - \sqrt{2})]^{\mathbf{2}} = \frac{1}{16}\left(6 + 2 - 2\sqrt{12}\right) = \frac{1}{16}\left(8 - 2\sqrt{12}\right) = \frac{1}{2} - \frac{2\sqrt{3}}{16}$$

$$\frac{1}{2} - \frac{\sqrt{3}}{8} = \frac{\mathbf{2} - \sqrt{\mathbf{3}}}{\mathbf{4}}$$

Aplicamos la fórmula:

$$\mathrm{sen}^{\mathbf{2}}\mathrm{a} + \cos^{\mathbf{2}}\mathrm{a} = 1$$

Y obtenemos el valor de cos a

$$\cos^{\mathbf{2}}\mathrm{a} = 1 - \mathrm{sen}^{\mathbf{2}}\mathrm{a}$$

$$\cos^2\mathrm{a} = 1 - \frac{2 - \sqrt{3}}{4} = \frac{4 - 2 + \sqrt{3}}{4} = \frac{\mathbf{2} + \sqrt{\mathbf{3}}}{\mathbf{4}}$$

Aplicamos la fórmula tga=sena/cos a

$$\mathrm{tg}^2\mathrm{a} = \frac{\mathrm{sen}^{\mathbf{2}}\mathrm{a}}{\cos^{\mathbf{2}}\mathrm{a}} = \frac{\frac{2 - \sqrt{3}}{4}}{\frac{2 + \sqrt{3}}{4}} = \frac{2 - \sqrt{3}}{2 + \sqrt{3}} = \frac{(2 - \sqrt{3})}{(2 + \sqrt{3})}\frac{(2 - \sqrt{3})}{(2 - \sqrt{3})} = \frac{(2 - \sqrt{3})^{\mathbf{2}}}{4 - 3}$$

$$= (2 - \sqrt{3})^{\mathbf{2}} = 4 + 3 - 4\sqrt{3} = \mathbf{7} - \mathbf{4}\sqrt{\mathbf{3}}$$

$\mathrm{tg}^2\mathrm{a} = (2-\sqrt{3})^{\mathbf{2}}$

Luego, extrayendo la raíz cuadrada

$tga \ = 2-\sqrt{3} \ \ (1)aclaración$

Aplicamos la fórmula:

$$tg2\mathrm{a} = \frac{2tga}{1-\mathrm{tg}^2\mathrm{a}}$$

$$tg2\mathrm{a} = \frac{2(2-\sqrt{3}\,)}{1-7-4\sqrt{3}} = \frac{2(2-\sqrt{3}\,)}{1-7+4\sqrt{3}} = \frac{2(2-\sqrt{3}\,)}{-6+4\sqrt{3}} = \frac{2(2-\sqrt{3}\,)}{4\sqrt{3}-6}$$

$$\frac{2(2-\sqrt{3}\,)}{(4\sqrt{3}-6)}\,\frac{(4\sqrt{3}+6)}{(4\sqrt{3}+6)} = \frac{2(8\sqrt{3}-4\sqrt{3}\sqrt{3}+12-6\sqrt{3})}{16x3-36} =$$

$$\frac{2(8\sqrt{3}-12+12-6\sqrt{3})}{48-36} = \frac{2x2x\sqrt{3}}{12} = \frac{4\sqrt{3}}{12} = \frac{\sqrt{3}}{3}$$

Aplicamos la fórmula:

$$\boldsymbol{cotg2a} = \frac{1}{tg2a} = \frac{1}{\frac{\sqrt{3}}{3}} = \frac{3}{\sqrt{3}} = \frac{3\sqrt{3}}{3} = \sqrt{\mathbf{3}}$$

Aclaración:

Si no nos damos cuenta de ese detalle se puede resolver partiendo del resultado final

$\mathrm{tg}^2\mathrm{a} = 7-4\sqrt{3}$

$tga \ = \sqrt{7-4\sqrt{3}}$

Este radical lo podemos transformar en la suma de dos radicales cuadráticos.

$\sqrt{7-4\sqrt{3}} = \sqrt{\mathrm{x}}-\sqrt{\mathrm{y}}$

$$\sqrt{7-4\sqrt{3}}^{\,2} = (\sqrt{x} - \sqrt{y})^2$$

$$7 - 4\sqrt{3} = x + y - \sqrt{4xy}$$

$$7 = \ x + y$$

$$-4\sqrt{3} = -\sqrt{4xy}$$

$$(-4\sqrt{3})^2 = \ (-\sqrt{4xy})^2$$

$$48 = \ 4xy$$

$$12 = \ xy$$

Por tanto, tenemos las dos ecuaciones:

$$7 = \ x + y \ \ ecuación\ 1$$

$$12 = \ xy \quad ecuación\ 2$$

Despejamos x de la ecuación 2:

$$x = \ \frac{12}{y}$$

Sustituyendo en la ecuación 1:

$$7 = \frac{12}{y} + y$$

$$y^2 - 7y + 12 = 0$$

$$y = \ \frac{7 \pm \sqrt{49-48}}{2} = \frac{7 \pm \sqrt{1}}{2} = \ \frac{7 \pm 1}{2}$$

$$y_1 = \ \frac{7+1}{2} = \ 8$$

$$y_2 = \ \frac{7-1}{2} = \ 6$$

Para y=3

$$\boldsymbol{x} = \frac{12}{3} = \mathbf{4}$$

Luego,

$$\sqrt{7 - 4\sqrt{3}} = \sqrt{\mathrm{x}} - \sqrt{\mathrm{y}}$$

$$\sqrt{7 - 4\sqrt{3}} = \sqrt{4} - \sqrt{3} \ = \mathbf{2} - \sqrt{\mathbf{3}}$$

Valor de la tga:

$$tga \ = 2 - \sqrt{3} \ \ (1)$$