PROBLEMAS DE TRIGONOMETRÍA

Problema 27:

Sabiendo que

$$sen\ a = \frac{1}{2}\sqrt{2-\sqrt{2}}$$

Hallar tg 2a

Solución Problema 27:

$$sen\ a = \frac{1}{2}\sqrt{2-\sqrt{2}}$$

La fórmula que vamos a emplear para calcular tga 2ª es:

$$tg\ 2a = \frac{2tga}{1-tg^2a}$$

Para ello, vamos a calcular el coseno de a, partiendo del valor del seno a y de la fórmula fundamental de trigonometría

$$sen^2a + cos^2a = 1$$

Así:

$$cos^2a = 1 - sen^2a = 1 - [\frac{1}{2}\sqrt{2-\sqrt{2}]}^2 = 1 - \left[\frac{1}{4}\left(2-\sqrt{2}\right)\right] =$$

$$1 - \left[\frac{2}{4} - \frac{\sqrt{2}}{4}\right] = 1 - \left(\frac{1}{2} - \frac{\sqrt{2}}{4}\right) = 1 - \left(\frac{2-\sqrt{2}}{4}\right) = \frac{4-2+\sqrt{2}}{4}$$

$$\frac{2+\sqrt{2}}{4}$$

$$\boldsymbol{cos^2a = \frac{2+\sqrt{2}}{4}}$$

$$cosa = \sqrt{\frac{2+\sqrt{2}}{4}} = \frac{1}{2}\sqrt{2+\sqrt{2}}$$

A continuación calculamos sen^{2}a. partiendo del valor inicial del seno de a:

$$sen^2a = \left[\frac{1}{2}\sqrt{2-\sqrt{2}}\right]^2 = \frac{1}{4}\left(2-\sqrt{2}\right) = \frac{2-\sqrt{2}}{4}$$

Ahora aplicando la fórmula del ángulo doble de la tg, tenemos:

$$tg\ 2a = \frac{2tga}{1-tg^2a} = \frac{2\frac{sena}{cosa}}{1-(\frac{sena}{cosa})^2}$$

$$tg\ 2a = \frac{2\frac{\cancel{\frac{1}{2}}\sqrt{2-\sqrt{2}}}{\cancel{\frac{1}{2}}\sqrt{2+\sqrt{2}}}}{1-\frac{sen^2a}{cos^2a}} = \frac{\frac{2\sqrt{2-\sqrt{2}}}{\sqrt{2+\sqrt{2}}}}{\frac{cos^2a-sen^2a}{cos^2a}} =$$

$$= \frac{\frac{2\sqrt{2-\sqrt{2}}}{\sqrt{2+\sqrt{2}}}}{\frac{\frac{2+\sqrt{2}}{4}-\frac{2-\sqrt{2}}{4}}{\frac{2+\sqrt{2}}{4}}} = \frac{\frac{2\sqrt{2-\sqrt{2}}}{\sqrt{2+\sqrt{2}}}}{\frac{\frac{2+\sqrt{2}-2+\sqrt{2}}{4}}{\frac{2+\sqrt{2}}{4}}} = \frac{\frac{2\sqrt{2-\sqrt{2}}}{\sqrt{2+\sqrt{2}}}}{\frac{\frac{\cancel{2}+\sqrt{2}\cancel{-2}+\sqrt{2}}{4}}{\frac{2+\sqrt{2}}{4}}}$$

$$= \frac{\frac{2\sqrt{2-\sqrt{2}}}{\sqrt{2+\sqrt{2}}}}{\frac{\frac{2\sqrt{2}}{\cancel{4}}}{\frac{2+\sqrt{2}}{\cancel{4}}}} = \frac{\frac{2\sqrt{2-\sqrt{2}}}{\sqrt{2+\sqrt{2}}}}{\frac{2\sqrt{2}}{2+\sqrt{2}}} = \frac{\frac{2\sqrt{2-\sqrt{2}}}{\cancel{\sqrt{2+\sqrt{2}}}}}{\frac{2\sqrt{2}}{\cancel{(\sqrt{2+\sqrt{2}})}(\sqrt{2+\sqrt{2}})}} =$$

$$\frac{\not{2}(\sqrt{2-\sqrt{2}})(\sqrt{2+\sqrt{2}})}{\not{2}\sqrt{2}} = \frac{(\sqrt{2-\sqrt{2}})(\sqrt{2+\sqrt{2}})}{\sqrt{2}} =$$

$$\frac{(\sqrt{2-\sqrt{2}})(\sqrt{2+\sqrt{2}})}{\sqrt{2}} = \frac{\sqrt{(2-\sqrt{2})(2+\sqrt{2})}}{\sqrt{2}} =$$

$$\frac{\sqrt{4-2}}{\sqrt{2}} = \frac{\sqrt{2}}{\sqrt{2}} = 1$$

$$\boldsymbol{tg\ 2a = \pm 1}$$