PROBLEMAS DE EXPRESIONES ALGEBRÁICAS Y OPERACIONES

Problema 45:

Efectuar las operaciones siguientes:

A) $\left(a^{\frac{7}{2}} - a^{3} + a^{\frac{5}{2}} - a^{2} + a^{\frac{3}{2}} - a + a^{\frac{1}{2}} - 1 \right)(a^{\frac{1}{2}} + 1)$

B) $\left(\frac{ay}{x}\right)^{\frac{1}{2}} x \left(\frac{bx}{y^2}\right)^{\frac{1}{3}} x \left(\frac{y^2}{a^2 b^2}\right)^{\frac{1}{3}}$

Solución Problema 45:

A) $\left(a^{\frac{7}{2}} - a^{3} + a^{\frac{5}{2}} - a^{2} + a^{\frac{3}{2}} - a + a^{\frac{1}{2}} - 1 \right)(a^{\frac{1}{2}} + 1)$

$$\left[\left(\sqrt{a^{7}} + \sqrt{a^{5}} + \sqrt{a^{3}} + \sqrt{a}\right) - (a^{3} + a^{2} + a + 1)\right](\sqrt{a} + 1)$$

$$\left[\left(a^{3}\sqrt{a} + a^{2}\sqrt{a} + a\sqrt{a} + \sqrt{a}\right) - (a^{3} + a^{2} + a + 1)\right](\sqrt{a} + 1)$$

$$\left[\sqrt{a}(a^{3} + a^{2} + a + 1) - (a^{3} + a^{2} + a + 1)\right]\left(\sqrt{a} + 1\right) =$$

$$\left[\left(a^{3} + a^{2} + a + 1\right)(\sqrt{a} - 1)\right]\left(\sqrt{a} + 1\right) =$$

$$\left(a^{3} + a^{2} + a + 1\right)(\sqrt{a} - 1)\left(\sqrt{a} + 1\right) =$$

$$(a^{3} + a^{2} + a + 1)(a - 1)$$

$$= a^{4} + a^{3} + a^{2} + a - a^{3} - a^{2} - a - 1 = \mathbf{a^{4} - 1}$$

B)$\left(\frac{ay}{x}\right)^{\frac{1}{2}}\text{x}\left(\frac{bx}{y^2}\right)^{\frac{1}{3}}\text{x}\left(\frac{y^2}{a^2b^2}\right)^{\frac{1}{4}}$

$$\left(\frac{ay}{x}\right)^{\frac{1}{2}}\text{x}\left(\frac{bx}{y^2}\right)^{\frac{1}{3}}\text{x}\left(\frac{y^2}{a^2b^2}\right)^{\frac{1}{4}} = \sqrt{\frac{ay}{x}}\sqrt[3]{\frac{bx}{y^2}}\sqrt[4]{\frac{y^2}{a^2b^2}} = \sqrt{\frac{ay}{x}}\sqrt[3]{\frac{bx}{y^2}}\sqrt{\frac{y}{ab}}$$

$$\sqrt{\frac{\cancel{a}y^2}{\cancel{a}bx}}\sqrt[3]{\frac{bx}{y^2}} = \sqrt{\frac{y^2}{bx}}\sqrt[3]{\frac{bx}{y^2}} = \frac{y}{\sqrt{bx}}\sqrt[3]{\frac{bx}{y^2}} = \frac{y\sqrt{bx}}{bx}\sqrt[3]{\frac{bx}{y^2}} = y\sqrt{bx}\sqrt[3]{\frac{bx}{b^3x^3y^2}}$$

$$y\sqrt{bx}\sqrt[3]{\frac{1}{b^2x^2y^2}} = y\sqrt{bx}\,\frac{\sqrt[3]{\text{bxy}}}{\sqrt[3]{b^2x^2y^2}\sqrt[3]{\text{bxy}}} = \frac{y\sqrt{bx}\sqrt[3]{\text{bxy}}}{\sqrt[3]{b^3x^3y^3}} =$$

$$\frac{\cancel{y}\sqrt{bx}\sqrt[3]{\text{bxy}}}{\text{xb}\cancel{y}} = \frac{\sqrt{bx}\sqrt[3]{\text{bxy}}}{\text{xb}} = \frac{\cancel{\sqrt{bx}}\sqrt[3]{\text{bxy}}}{\cancel{\sqrt{bx}}\sqrt{bx}} = \frac{\sqrt[3]{\text{bxy}}}{\sqrt{bx}} = \sqrt[6]{\frac{b^2x^2y^2}{b^3x^3}}$$

$$\sqrt[6]{\frac{y^2}{\text{bx}}} = \frac{\sqrt[3]{\mathbf{y}}}{\sqrt[6]{\mathbf{bx}}} = \frac{\mathbf{y}^{\frac{1}{3}}}{(\mathbf{bx})^{\frac{1}{6}}}$$